

Self-resonant plasma wake-field excitation by a laser pulse with a steep leading edge for particle acceleration

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The self-modulational instability of a relatively long laser pulse with a power close to or less than the critical power for relativistic self-focusing in plasma is considered. Strong wake-field excitation occurs as the result of a correlated transverse and longitudinal evolution of the pulse. The dependence of the magnitude of the plasma wave on the duration of a flat-top pulse is investigated. The power necessary to reach 20% electron density modulation behind the laser pulse is shown to decrease as the pulse duration increases, while the phase velocity of the plasma wave remains close to the group velocity of the laser pulse. This provides an opportunity to operate a laser wake-field accelerator in the self-modulated regime at a subcritical laser power, at least twice less than the critical one, and to obtain a sufficiently large accelerating gradient (> 20 GV/m) in a region which is longer than that required for the acceleration of an ultrarelativistic particle.

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I. INTRODUCTION

During the past decade, plasma-based accelerator concepts have attracted much attention, in view of the possibility to reach ultrahigh accelerating fields exceeding 1 GV/m. Various plasma-based accelerator schemes are currently under investigation [1]. The most elaborate one is the plasma beat-wave acceleration (PBWA) concept [2]. This concept relies on the resonant excitation of a Langmuir wave in a homogeneous plasma. Recent experiments with CO₂ lasers at University of California at Los Angeles have demonstrated acceleration gradients of about 2 GV/m [3], but the very high degree of uniformity of the background plasma required for resonant excitation is a serious disadvantage of the PBWA scheme.

The alternative concept, referred to as the laser wake-field accelerator (LWFA), does not involve a highly uniform plasma [4]. It relies on the shock excitation of a plasma wave by a short and intense laser pulse propagating through the plasma. In standard LFWA, efficient wake generation requires the pulse duration τ to be shorter than half a period of the plasma electrons' oscillation [4],

$$\tau < \pi / \omega_p ,$$

where $\omega_p \equiv (4\pi e^2 n_0 / m)^{1/2}$ is the plasma frequency and n_0 is the plasma density. For a given plasma frequency, however, the amplitude of the electric field E_z in a linear Langmuir wave is limited:

$$E_z < m \omega_p c / e .$$

As the plasma frequency, from the above arguments, is limited with the inverse pulse duration, $\omega_p < \pi / \tau$, one sees that even for a very intense laser pulse the highest

accelerating gradient possible with the LWFA scheme is restricted by technological limitations on the laser pulse duration:

$$E_z < \pi m c / e \tau$$

(the limitation is not very tight, though: say, $\tau = 100$ fs corresponds to $E_z < 55$ GV/m).

A promising way to combine the advantages of the above schemes and to avoid their limitations is the recently proposed concept of *self-modulated* laser wake-field acceleration (SMLWFA) [5]. This concept makes use of the natural nonlinear pulse dynamics. Namely, if (i) the duration of the laser pulse is longer than several plasma oscillation periods and (ii) the peak laser power is close to the critical power for relativistic self-focusing [6],

$$P_c \simeq 16.2 (\omega_0 / \omega_p)^2 \text{ GW} , \quad (1)$$

where ω_0 is the laser carrier frequency, the pulse appears to be unstable against plasma density perturbations with frequency ω_p and a phase velocity close to the group velocity of the laser pulse. The mechanism of this instability can be understood in terms of refractive plasma properties [7]. Because of the density dependence of the refractive index of the plasma, each region (in the comoving frame of reference) of decreased density acts as a focusing lens while each region of increased density causes defocusing. Therefore, plasma density perturbations generated by the leading edge of the pulse give rise to periodic focusing and defocusing of the main body of the pulse. In turn, the modulation of the laser pulse leads to a resonant enhancement of the plasma density modulation, thus forming a positive feedback for pulses longer than several plasma wavelengths. As a result, the initially smooth laser pulse excites a strong plasma wake field, in

which an enhanced acceleration of relativistic electrons is possible. Another approach to the description of the pulse self-modulation can be introduced in terms of the theory of forward and near-forward stimulated Raman scattering (see [8–11]).

For given laser pulse parameters, both conditions (i) and (ii) can be satisfied by choosing a sufficiently high plasma density. The process of plasma wave excitation appears then to be very efficient. Theoretical considerations and numerical simulations [5,9,12] predict accelerating gradients of the order of 20–100 GV/m, which, for the same laser pulse parameters, provide more than an order of magnitude enhancement of particle acceleration as compared to the standard LWFA scheme [5,12].

Recent proof-of-principle experiments [3,13,14] have demonstrated the possibility of particle acceleration using all the schemes mentioned. The most promising concept with the greatest experimentally observed value of the accelerating gradient (~ 30 GV/m) is SMLWFA [13]. However, the experiments also revealed the general problem for all laser-based acceleration schemes: the length of the region where acceleration is possible, is strongly limited. First of all, there is a natural length scale (the so-called acceleration length l_a [2]) associated with the time needed for an ultrarelativistic particle to slip over a plasma half-wavelength,

$$l_a = \pi \gamma_p^2 c / \omega_p, \quad (2)$$

where $\gamma_p = \omega_0 / \omega_p$ is the Lorentz factor corresponding to the phase velocity of the plasma wave. To obtain a longer time of acceleration, one should use a higher γ_p factor (that is, a more rarefied plasma). The second limitation comes from the laser pulse power available; trying to reach higher electric fields by focusing, one inevitably decreases the length of the region where a strong accelerating field exists. The length of this region is determined by the Rayleigh length $Z_R = L_\perp^2 \omega_0 / 2c$, where L_\perp is the characteristic transverse size of the pulse. Self-focusing and relativistic waveguiding can enlarge the size of this region but not considerably (see [12]). If either the acceleration length l_a or the Rayleigh length Z_R is small, then the energy gain of the accelerated particles may be relatively low, despite extremely high accelerating gradients inside the plasma.

As the SMLWFA concept was shown to provide the highest accelerating fields, the duration of the self-modulation stage is a problem of considerable interest. Previous numerical studies of self-modulation in a relatively dense plasma ($\gamma_p \simeq 20$) for a laser power close to [5] and above [12] the critical one have shown that the strong wake field appearing at the self-modulation stage can be excited in a region that is just about l_a in length. To reach a final particle energy in the GeV range, one should use a more dilute plasma corresponding to higher γ_p factors ($\gamma_p \geq 50$) and, consequently, a greater l_a . There is presently no understanding whether it is possible, in this case, to have long enough regions (about l_a) of strong wake-field excitation in the SMLWFA scheme

without the need for transverse profiling of the initial plasma density to provide pulse guiding.

II. NUMERICAL MODEL

In the present paper we consider the self-resonant excitation of a plasma wake field by a relatively long laser pulse with a steep leading edge. The efficiency of plasma wave excitation is studied as a function of the pulse duration L and its power P . Because the amplitude of the plasma wave behind the bunch depends on both its power and its duration, one may hope to optimize the scheme with respect to these parameters. Special attention is given to an assessment of the length of the region where strong wave-field excitation is possible. Stimulated Raman backscattering (SRBS) is not included in the numerical model; its influence is discussed in the conclusions.

Our numerical model is based on the set of envelope equations [5] for the evolution of an axisymmetric laser pulse and the electron density modulation in a rarefied plasma ($\omega_p \ll \omega_0$),

$$2i\omega_0 \frac{\partial a}{\partial t} + c^2 \Delta_\perp a + \frac{\omega_p^2}{4} |a|^2 a = \omega_p^2 \nu a, \quad (3)$$

$$v_g^2 \frac{\partial^2 \nu}{\partial \xi^2} + \omega_p^2 \nu = \frac{c^2}{4} \Delta |a|^2, \quad (4)$$

where $a = eE_0 / m\omega_0 c$ is the normalized amplitude of the electromagnetic wave, $\nu \equiv \delta n / n_0$ is the normalized electron density perturbation, and $\xi = Z - v_g t$ is the longitudinal coordinate in a frame of reference moving at the group velocity $v_g = c \sqrt{1 - (\omega_p / \omega_0)^2}$. In the above equations $\Delta \equiv \Delta_\perp + \partial^2 / \partial \xi^2$ is the Laplace operator and Δ_\perp is its transverse part; in the axisymmetric case Δ_\perp reduces to $\Delta_\perp = r^{-1} (\partial / \partial r) (r \partial / \partial r)$, where r is the transverse coordinate. Both equations are valid in the weakly nonlinear limit $|a|^2 < 1$, $|\nu| < 1$. The plasma ions are assumed to be immobile; the pulse duration is thus assumed to be sufficiently short to neglect a possible ion displacement. Both one-dimensional (1D) and 3D-axisymmetric resonant-instability effects can be described in terms of these equations [7–9].

The above set of equations is investigated numerically under the assumption that there are no density and field perturbations ahead of the pulse and under the condition that a and ν decay as $r \rightarrow \infty$. The laser pulse is assumed to be focused onto the plasma boundary so that the phase front of the pulse is initially planar. As for the initial shape of the pulse, it is assumed to be Gaussian in the transverse direction and flat-top with steep edges in the longitudinal direction,

$$a(t=0, \xi, r) = a_0 \exp(-r^2 / L_\perp^2) f(\xi), \quad (5)$$

where

$$f(\xi) = \begin{cases} \exp[-(\xi - L_\parallel)^4 / L_f^4] & \text{for } \xi > L_\parallel, \\ 1 & \text{for } 0 < \xi < L_\parallel, \\ \exp(-\xi^4 / L_f^4) & \text{for } \xi < 0, \end{cases} \quad (6)$$

with the parameters L_\perp , L_\parallel , and L_f characterizing the

TABLE I. Laser and plasma parameters used in calculations.

Parameter	Value
Laser parameters	
Wavelength λ_0	1.06 μm
Initial pulse radius L_\perp	106 μm
Rayleigh length Z_R	3.3 cm
Front duration L_f	12 μm ($L_f/c \approx 40$ fs)
Flattop duration L_\parallel	60–300 μm ($L_\parallel/c \approx 0.2$ –1.0 ps)
Pulse power P	4–10 TW
Energy per pulse I_0	1–5 J
Plasma parameters	
Electron density n_0	2×10^{18} cm^{-3}
Plasma wavelength λ_p	24 μm
Critical power P_c	8.6 TW

transverse dimension, the flat-top length, and the length of the leading edge, respectively.

The flat-top length L_\parallel was the input parameter we varied in our calculations while L_\perp and L_f remained fixed. Another variable was the initial pulse amplitude a_0 that enabled us to vary the peak laser power P while keeping the radius of the focal spot L_\perp fixed. In the present paper, laser powers in the critical and in the sub-critical regime ($0.5 < P/P_c < 1.1$) are considered [see Eq. (1)].

Modern laser technology can provide a peak power of several TW in the micrometer range of wavelengths [15]. One may easily estimate from Eq. (1) that this power is close to the critical one for a plasma density about 10^{18} cm^{-3} . To be specific, we consider a Nd-glass laser (wavelength $\lambda_0 = 1.06$ μm) and a plasma density $n_0 = 2 \times 10^{18}$ cm^{-3} . The critical power is then $P_c \approx 8.6$ TW and the plasma wavelength is $\lambda_p \equiv 2\pi v_g / \omega_p \approx 24$ μm . To provide a relatively long diffraction length and efficient plasma wave excitation, the focal spot size and the pulse front duration were chosen to be $L_\perp = 100\lambda_0 = 106$ μm and $L_f = 12$ μm ($L_f/c \approx \pi/\omega_p = 40$ fs). The Rayleigh length is then $Z_R \approx 3.3$ cm, that is well in excess of the acceleration length $l_a \approx 0.7$ cm. The laser and plasma parameters used in calculations are summarized in Table I. The results of the computations are shown in Figs. 1–6.

III. NUMERICAL RESULTS

Figure 1(a) illustrates the instability of a long laser pulse against self-modulation: shown is the dependence of the electron density at the axis of the system on the ξ coordinate. The figure corresponds to the moment when the amplitude of the plasma wave behind the pulse is at its maximum. The initial peak power of the laser pulse is 5 TW $\approx 0.6P_c$, the pulse duration is 0.7 ps $\approx 9\lambda_p$, and the pulse penetration into the plasma is 2.7 cm. One can see a nearly exponential growth of the intensity of the plasma wave from the head of the pulse to its tail. Behind the pulse the amplitude of the oscillation remains constant because no absorption mechanism is included in the model.

However, the exponential growth of the plasma wave

with ξ coordinate seen in Fig. 1(a) appears to take place only at relatively early stages of the pulse penetration into the plasma. Later on, the amplitude of the plasma wave seems to saturate. This is illustrated in Fig. 1(b) where we show the electron density modulation at the axis of the system for the same pulse parameters as in Fig. 1(a) but at a penetration depth of 4.6 cm. At about 100 – 120 μm behind the pulse head, the growth of the density modulation, initially close to the exponential one, starts to slow down. As a result, the magnitude of the plasma wave behind the pulse is approximately twice lower than in Fig. 1(a).

The wake-field amplitude thus depends strongly on the distance traveled by the pulse in the plasma [5,7,9,12]. The solid line in Fig. 2 shows the amplitude of the electron density oscillations behind the laser pulse as a function of its penetration Z into the plasma (pulse parameters are the same as in Fig. 1). The density modulation is seen to reach its maximum value at a depth of about 2.7 cm.

As was mentioned above, both the amplitude and the phase velocity of the plasma wave are relevant param-

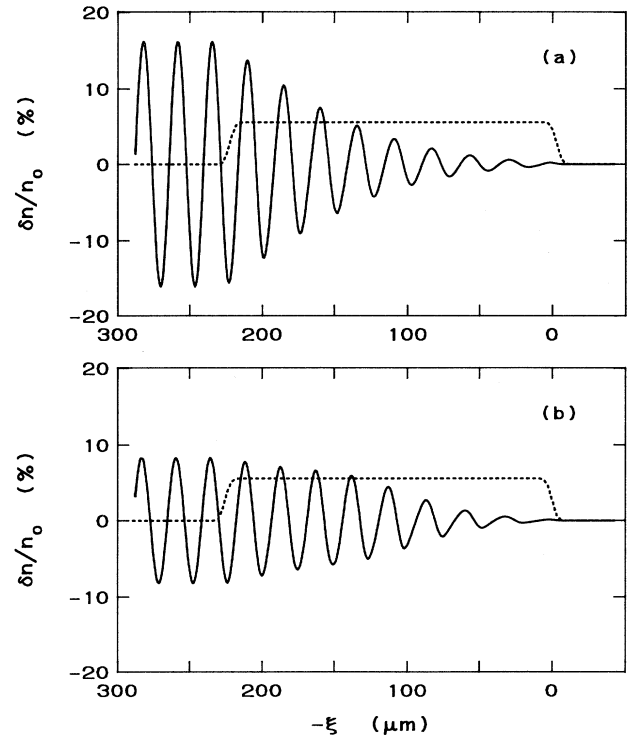


FIG. 1. The electron density modulation (solid line) at the axis of a long laser pulse for the stage of developed pulse self-modulation: (a) at a penetration depth of 2.7 cm (corresponding to the maximum of the density modulation); (b) at a penetration depth of 4.6 cm. The pulse has a length of about 200 μm (its initial shape is represented by the dotted line) and moves from the left to the right; the initial pulse power is 5 TW $\approx 0.6P_c$. A nearly exponential growth of the intensity of the plasma wave from the head of the pulse to its tail is clearly seen in (a), while in (b) the growth of the density modulation noticeably slows down at 100 – 120 μm behind the pulse head.

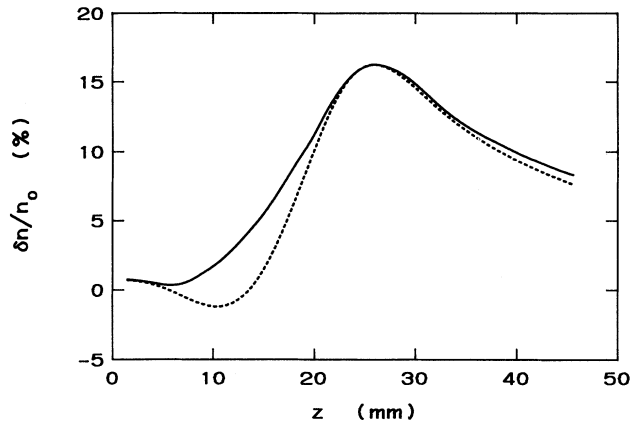


FIG. 2. The amplitude of the electron density oscillation behind the laser pulse (solid line) as a function of its penetration depth into the plasma. The pulse parameters are the same as in Fig. 1. The density modulation is seen to reach its maximum value at a depth of about 2.7 cm. The dotted line shows $\delta n/n_0$ at a fixed point behind the pulse in the comoving frame of reference ($\xi = -235 \mu\text{m} = \text{const}$). The difference between the solid line and the dotted line indicates the influence of the phase slippage of the plasma wave with respect to the laser pulse.

ters for particle acceleration. To illustrate the influence of the phase velocity, the dotted line in Fig. 2 shows the electron density at a fixed point behind the pulse in the comoving frame of reference ($\xi = -235 \mu\text{m} = \text{const}$); the point was chosen to coincide with the position of a maximum of the electron density at a depth of 2.7 cm. If the phase velocity of the plasma wave would be exactly equal to the group velocity of the laser pulse, the above point would coincide with a maximum of the electron density throughout the whole pulse path in the plasma, and the dotted line would never deviate from the solid one. The difference between the two lines is due to the phase shift of the plasma wave with respect to the comoving frame of reference. One may see that the phase velocity of the plasma wave is in fact different from the group velocity of the laser pulse, especially at the early stage of the pulse evolution ($5 \text{ mm} < Z < 15 \text{ mm}$). The difference is, however, negligibly small for $Z > 20 \text{ mm}$ and does not lead to a noticeable reduction of the length of the region where acceleration of an ultrarelativistic particle is possible.

The dependence of the peak density modulation on the pulse duration (for the same initial peak pulse power of 5 TW) is shown in Fig. 3; the pulse duration L is taken as FWHM, leading and trailing edges included: $L \approx L_{\parallel} + 1.53L_f$ (note that L_{\parallel} is varied in our calculations while L_f is always kept equal to $12 \mu\text{m}$). The peak magnitude of the plasma wave behind a laser pulse is seen to increase when the pulse length increases. Because of the nonlinear evolution of the pulse as it penetrates into the plasma, the peak density modulation occurs at different penetration depths for different pulse durations. Figure 4 shows the depth Z_{peak} at which the peak modulation takes place, as a function of the pulse duration. For longer pulses the peak modulation is seen to occur earlier

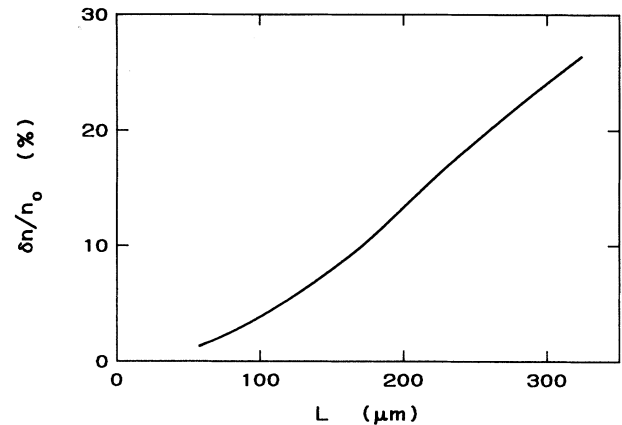


FIG. 3. The dependence of the peak density modulation on the laser pulse duration. For different pulse durations, the peak modulation occurs at different penetration depths into the plasma (see Fig. 4). The initial pulse power is $5 \text{ TW} \approx 0.6P_c$, the initial pulse radius is $L_{\perp} = 100\lambda_0 = 106 \mu\text{m}$.

than for short ones.

For particle acceleration, the most important parameters are the amplitude of longitudinal electric field behind the pulse and the length of the region of strong wake-field excitation. The accelerating field is proportional to the amplitude of the electron density modulation. Since our numerical model is limited to the range $\delta n/n_0 < 1$, we have chosen an electron density modulation of 20% as a value that is noticeably large, on the one hand, but still lies within the range of applicability of the model, on the other hand. For the plasma parameters used in our calculations, this value of the plasma density modulation corresponds to an accelerating field $E_z \sim 25 \text{ GV/m}$. The input laser power necessary to reach 20% modulation was studied as a function of the laser pulse duration. The results are shown in Fig. 5. The required laser power is seen to decrease as the pulse duration increases. One can

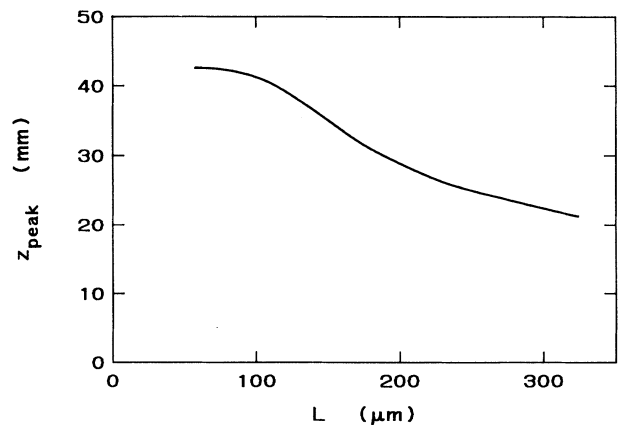


FIG. 4. The penetration depth Z_{peak} at which the peak density modulation takes place, as a function of the pulse duration. The pulse parameters are the same as in Fig. 3.

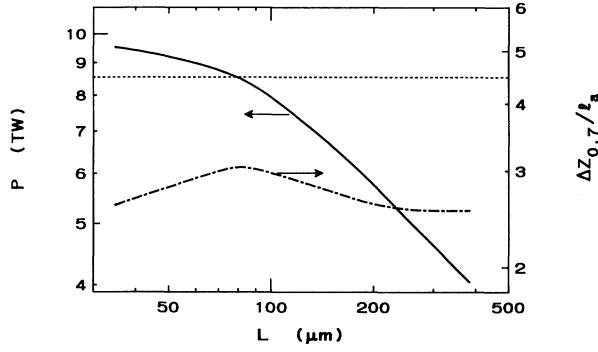


FIG. 5. The input laser power necessary to reach 20% modulation of the electron density behind the pulse, as a function of the pulse duration (solid line). The initial radius of the laser pulse is the same as in Fig. 3. The dashed line shows the critical power threshold. The length $\Delta Z_{0.7}$ of the region (in the laboratory frame) where the magnitude of the plasma wave behind the laser pulse exceeds 70% of its peak value is shown with the dotted line. $\Delta Z_{0.7}$ is measured in units of the acceleration length l_a ; it is seen to be 2.5–3 times longer than l_a throughout the whole range of pulse durations considered.

see a noticeable change in the slope of the curve as it crosses the critical power threshold (shown as the horizontal dashed line). For $P > P_c$ the dependence of the pulse power on the pulse length is weak, while for $P < P_c$ the power drops approximately as $L^{-1/2}$.

To characterize the length of the region of strong wake-field excitation, we define $\Delta Z_{0.7}$ as the distance (in the laboratory frame), over which the amplitude of the electron density oscillation behind the laser pulse remains above 70% of its peak value of 0.2 (that is, $\delta n/n_0 > 0.14$). The dot-dashed line in Fig. 5 shows $\Delta Z_{0.7}$ as a function of the pulse duration; $\Delta Z_{0.7}$ is normalized to the acceleration length l_a . For the pulse parameters under consideration, the length of the region where strong wake-field excitation exists is seen to weakly depend on the pulse duration and to be 2.5–3 times longer than the distance necessary for acceleration of an ultrarelativistic particle.

Finally, one of the key parameters in powerful laser operation is the total energy per pulse I_0 . The energy carried by the laser pulse (with the same parameters as in Fig. 5) is shown in Fig. 6. Being proportional to the product $P \times L$, the energy I_0 is seen to increase as the pulse duration increases. The reduction in the power required for longer pulses is therefore insufficient to lead to a reduction of the total pulse energy. For pulses longer than $\sim 100 \mu\text{m}$, the energy grows approximately as the square root of the pulse duration. However, in the range of the laser and plasma parameters considered, the pulse energy still remains within the present technology (below $\sim 5 \text{ J}$).

IV. DISCUSSION AND CONCLUSIONS

Before summarizing the results, it is worth mentioning that SRBS may turn out to be a potential problem for

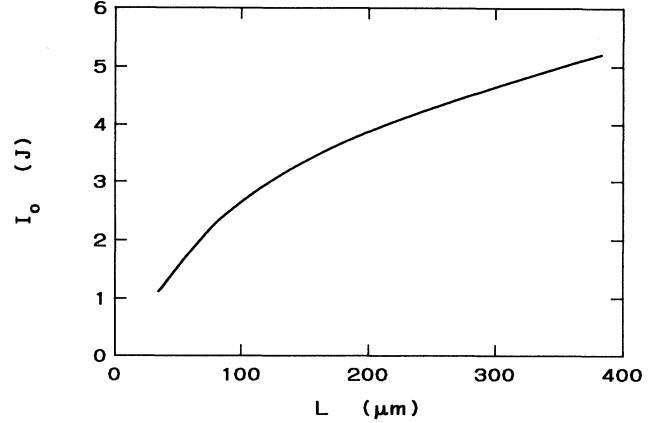


FIG. 6. The total energy carried by the laser pulse with the parameters shown in Fig. 5. For pulses longer than $\sim 100 \mu\text{m}$, the energy grows approximately as the square root of the pulse duration.

plasma wave excitation, especially in the case of long and powerful pulses. For SRBS to have a weak effect on the propagation of a nonmodulated laser pulse over a distance l , the scattered energy has to be much less than the total pulse energy. This is fulfilled if the inequality

$$a_0 k_0 L < \sqrt{2\omega_0/\omega_p} \times \ln \left\{ \frac{\pi}{2^{11/4}} a_0 (\omega_0/\omega_p)^{-5/4} \frac{mc^2}{T_e} \frac{L^2}{lr_e} (k_0 L)^{1/2} \right\} \quad (7)$$

holds [16]; here $r_e = e^2/(mc^2)$ and T_e is the plasma-electron temperature. One can see that for the parameters considered here (pulse duration less or about 1 ps, pulse power about one-half of the critical power, and $T_e \sim 10 \text{ eV}$) the nonequality (7) holds long enough for the self-modulation of the pulse to develop. In the latter stage of the pulse evolution SRBS is known to be suppressed [16]; hence, in our case SRBS is unimportant.

To conclude, the self-modulated regime of plasma wave excitation by a relatively long laser pulse with a steep leading edge was considered. For a given laser power and plasma density, the electron density modulation behind the pulse is shown to increase as the pulse duration increases. Respectively, the input laser power necessary to reach 20% modulation decreases as the pulse duration increases; in the subcritical region $P < P_c$ the power drops approximately as the square root of the pulse duration. The length of the region, where strong plasma field exists, depends weakly on the pulse duration, always remaining well in excess of the acceleration length l_a . For the largest pulse duration considered ($\sim 1 \text{ ps}$), it is

about 2.5 times larger than l_d . The SMLWFA concept thus provides an opportunity not only to operate at a subcritical power, at least twice less than the critical one, but also makes it possible, at the expense of usage of a longer pulse with higher total pulse energy, to obtain a sufficiently large wake field in a region that is fairly longer than that required for acceleration of an ultrarelativistic particle.

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